Universität Stuttgart
Institut für Wasser- und Umweltsystemmodellierung
Lehrstuhl für Hydromechanik und Hydrosystemmodellierung

Coupled Free and Porous-Medium Flow – SFB Project Area A

DuMuX Course 2018

July 20, 2018

Introduction

Motivation

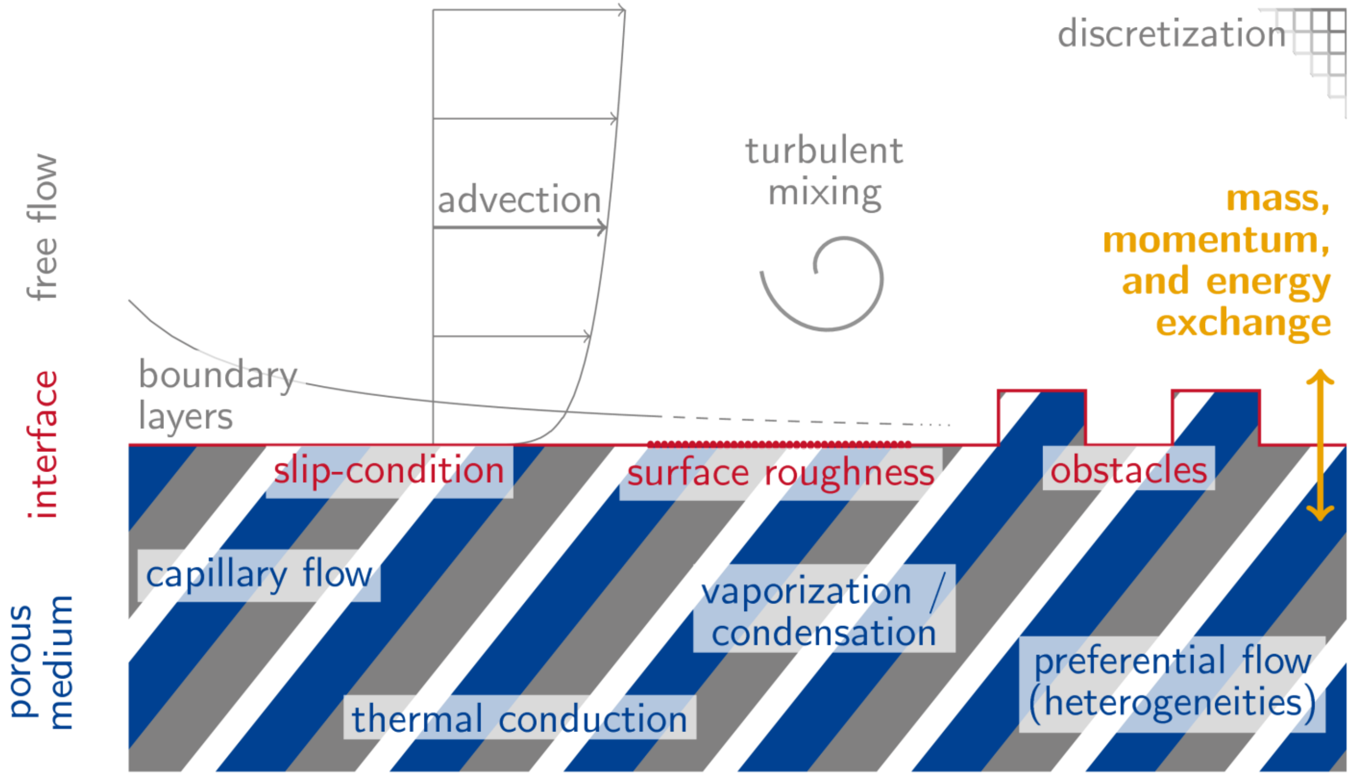
- urban climate
- CO_2 leakage
- atomic waste storage, …

- fuel cells
- heat exchanger
- oil filters, …

- soil water evaporation
- evaporative cooling
- soil salinization, …

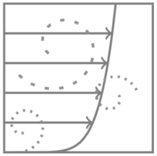
Introduction

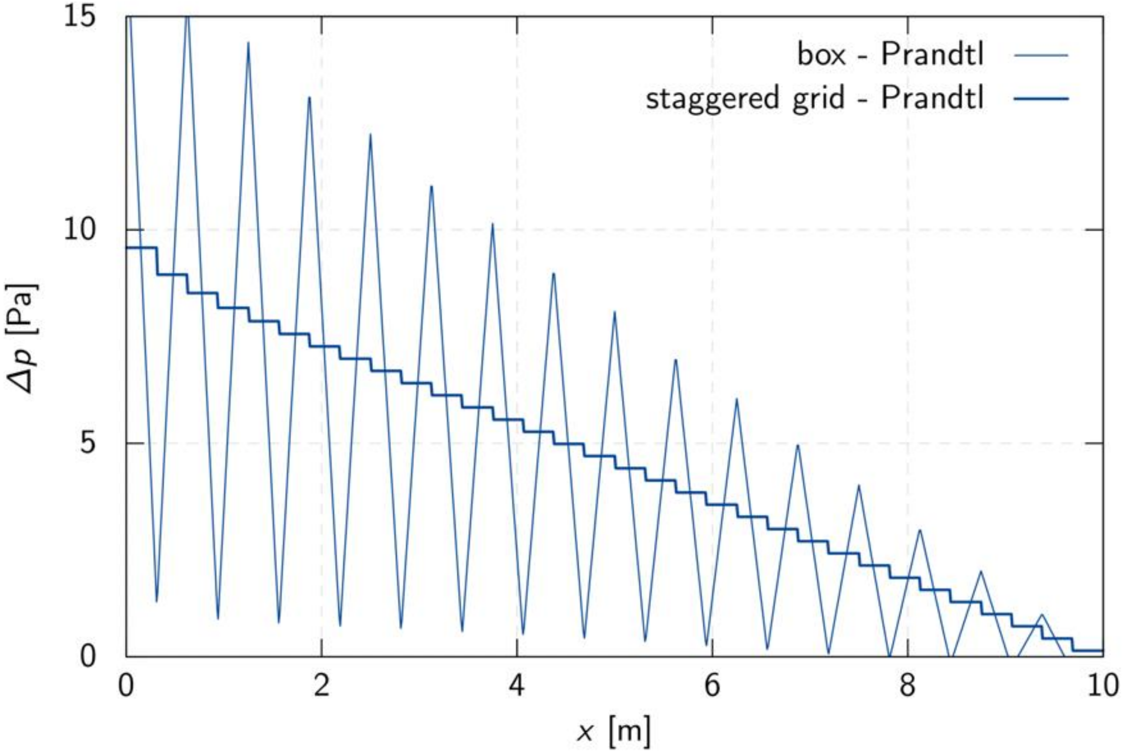
Processes and Properties



Introduction

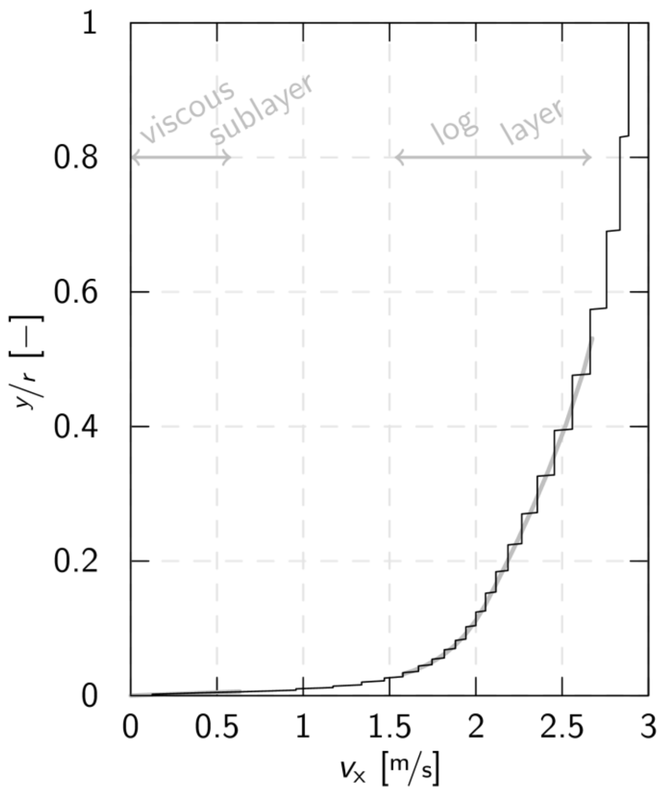
Discretization

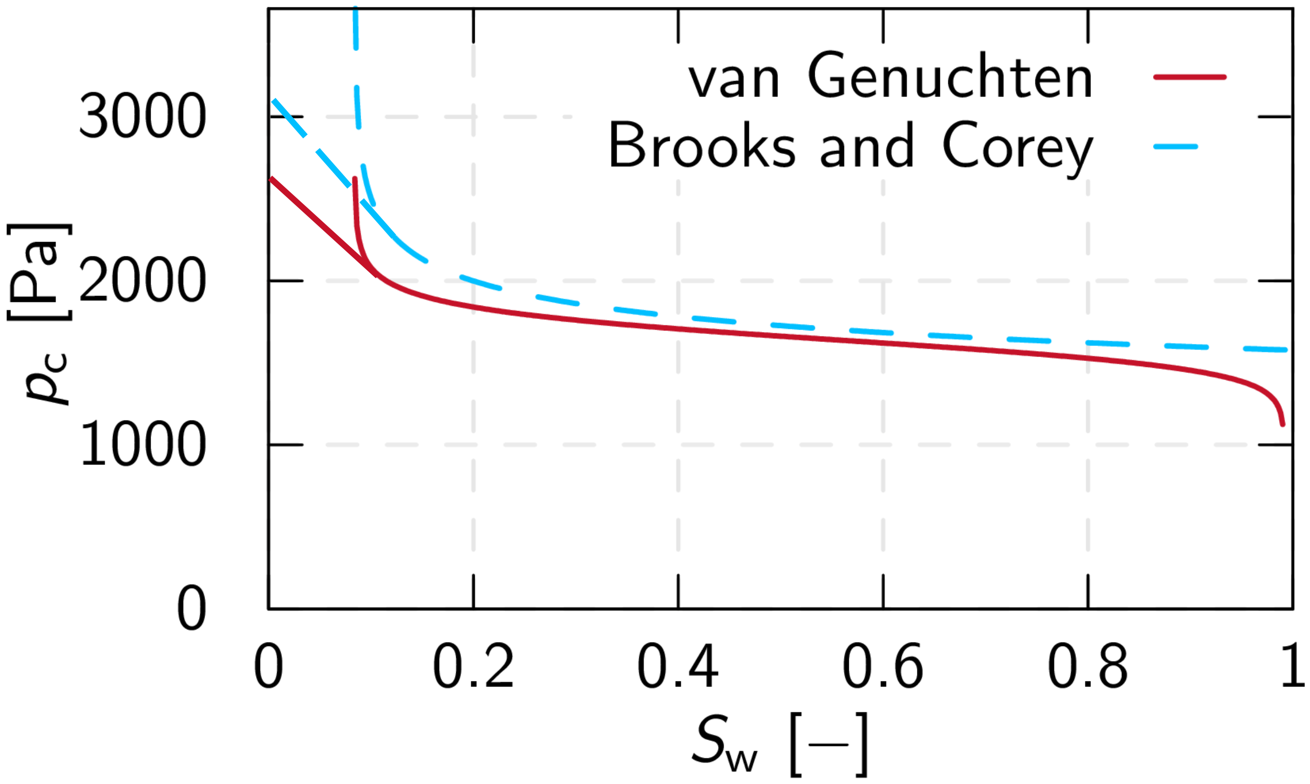




Introduction

Spatial Resolution of Processes




van Genuchten
Brooks and Corey
3000
2000
1000
0
0
0.2
0.4
0.6
0.8
1
p_c [Pa]
S_w [–]

Introduction

Model

Implementation

Results

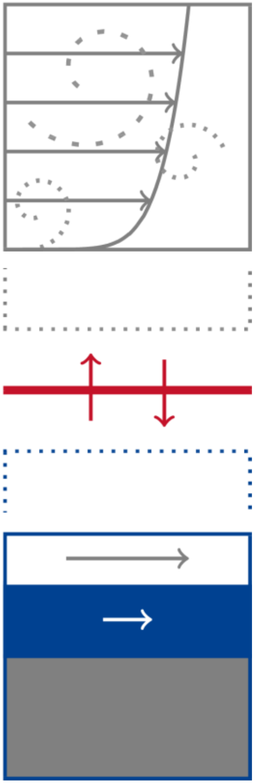
Model

Two-Domain/ Sharp-Interface Concept

- Stokes/ Navier-Stokes/ RANS
- 1 phase, n-components, non-isothermal

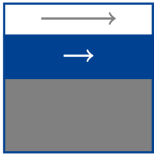
- no thickness, no storage
- local thermodynamic equilibrium
- continuity of fluxes
- continuity of state variables

- Darcy/ Forchheimer/ Richards
- m-phases, n-components, non-isothermal



Model

Porous Medium – Equations



- total mass balance

$$\sum_{\alpha\in\{\mathrm{l},\mathrm{g}\}}\left(\phi\frac{\partial\left(\varrho_{\mathrm{mol},\alpha}S_\alpha\right)}{\partial t}+\nabla\cdot\left(\varrho_{\mathrm{mol},\alpha}\mathbf{v}_\alpha\right)\right)=0$$

storage | advection

- component mass balance

$$\sum_{\alpha\in\{\mathrm{l},\mathrm{g}\}}\left(\phi\frac{\partial\left(\varrho_{\mathrm{mol},\alpha}S_\alpha x_\alpha^\kappa\right)}{\partial t}+\nabla\cdot\left(\varrho_{\mathrm{mol},\alpha}x_\alpha^\kappa\mathbf{v}_\alpha\right)+\nabla\cdot\mathbf{j}_\alpha^{\kappa,\mathrm{pm}}\right)=0$$

storage | advection | diffusion

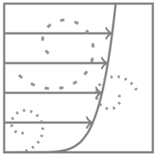
- energy balance

$$\sum_{\alpha\in\{\mathrm{l},\mathrm{g}\}}\left(\phi\frac{\partial\left(\varrho_\alpha S_\alpha u_\alpha\right)}{\partial t}+\nabla\cdot\left(\varrho_\alpha h_\alpha\mathbf{v}_\alpha\right)\right)+\left(1-\phi\right)\varrho_\mathrm{s}c_\mathrm{s}\frac{\partial T}{\partial t}-\nabla\cdot\left(\lambda^{\mathrm{pm}}\nabla T\right)=0$$

storage (fluids) | advection | storage (solid) | conduction

Model

Free Flow – Reynolds-Averaged Navier-Stokes Equations



- total mass balance

$$\frac{\partial \varrho_{\mathrm{mol,g}}}{\partial t} + \nabla \cdot \left(\varrho_{\mathrm{mol,g}} \bar{\mathbf{v}}_{\mathrm{g}} \right) = 0$$

storage — advection

- momentum balance

$$\frac{\partial \left(\varrho_{\mathrm{g}} \bar{\mathbf{v}}_{\mathrm{g}} \right)}{\partial t} + \nabla \cdot \left(\varrho_{\mathrm{g}} \bar{\mathbf{v}}_{\mathrm{g}} \bar{\mathbf{v}}_{\mathrm{g}}^{\mathsf{T}} \right) - \nabla \cdot \boldsymbol{\tau}_{\mathrm{eff}} + \nabla \cdot \left(\bar{p}_{\mathrm{g}} \mathbf{I} \right) - \varrho_{\mathrm{g}} \mathbf{g} = 0$$

storage — inertia — effective stress — pressure — gravity

- component mass balance

$$\frac{\partial \left(\varrho_{\mathrm{mol,g}} \bar{x}_{\mathrm{g}}^{\kappa} \right)}{\partial t} + \nabla \cdot \left(\varrho_{\mathrm{mol,g}} \bar{x}_{\mathrm{g}}^{\kappa} \bar{\mathbf{v}}_{\mathrm{g}} \right) + \nabla \cdot \mathbf{j}_{\mathrm{eff}}^{\kappa,\mathrm{ff}} = 0$$

storage — advection — effective diffusion

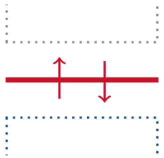
- energy balance

$$\frac{\partial \left(\varrho_{\mathrm{g}} \bar{u}_{\mathrm{g}} \right)}{\partial t} + \nabla \cdot \left(\varrho_{\mathrm{g}} \bar{h}_{\mathrm{g}} \bar{\mathbf{v}}_{\mathrm{g}} \right) + \sum_{\kappa \in \{\mathrm{w,a}\}} \nabla \cdot \left(\bar{h}_{\mathrm{g}}^{\kappa} \mathbf{j}_{\mathrm{mass,eff}}^{\kappa,\mathrm{ff}} \right) + \nabla \cdot \mathbf{j}_{\mathrm{cond,eff}}^{\mathrm{ff}} = 0$$

storage — advection — effective diffusion — effective conduction

Model

Interface – Coupling Conditions I



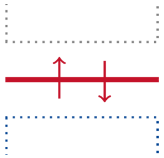
- total mass $$\left[\left(\varrho_{\text{mol,g}} \mathbf{v}_{\text{g}} \right) \cdot \mathbf{n} \right]^{\text{ff,if}} = - \left[\left(\varrho_{\text{mol,g}} \mathbf{v}_{\text{g}} + \varrho_{\text{mol,l}} \mathbf{v}_{\text{l}} \right) \cdot \mathbf{n} \right]^{\text{pm,if}}$$

- momentum (tangential) $$\left[\left(- \frac{\sqrt{K}}{\alpha_{\text{BJ}}} \left(\nabla \mathbf{v}_{\text{g}} \right) \mathbf{n} - \mathbf{v}_{\text{g}} \right) \cdot \mathbf{t}_i \right]^{\text{ff,if}} = 0$$

- momentum (normal) $$\left[\left(\left(\varrho_{\text{g}} \mathbf{v}_{\text{g}} \mathbf{v}_{\text{g}}^{\mathsf{T}} - \boldsymbol{\tau}_{\text{eff}} + p_{\text{g}} \mathbf{I} \right) \mathbf{n} \right) \cdot \mathbf{n} \right]^{\text{ff,if}} = p_{\text{g}}^{\text{pm,if}}$$

Model

Interface – Coupling Conditions II



- component mass

$$\left[x_\mathrm{g}^\kappa\right]^{\mathrm{ff,if}} = \left[x_\mathrm{g}^\kappa\right]^{\mathrm{pm,if}}$$

$$\left[\left(\varrho_{\mathrm{mol,g}} x_\mathrm{g}^\kappa \mathbf{v}_\mathrm{g} + \mathbf{j}_{\mathrm{eff}}^\kappa\right) \cdot \mathbf{n}\right]^{\mathrm{ff,if}}$$
$$= -\left[\left(\varrho_{\mathrm{mol,g}} x_\mathrm{g}^\kappa \mathbf{v}_\mathrm{g} + \varrho_{\mathrm{mol,l}} x_\mathrm{l}^\kappa \mathbf{v}_\mathrm{l} + \mathbf{j}_\mathrm{g}^\kappa + \mathbf{j}_\mathrm{l}^\kappa\right) \cdot \mathbf{n}\right]^{\mathrm{pm,if}}$$

- energy

$$[T]^{\mathrm{ff,if}} = [T]^{\mathrm{pm,if}}$$

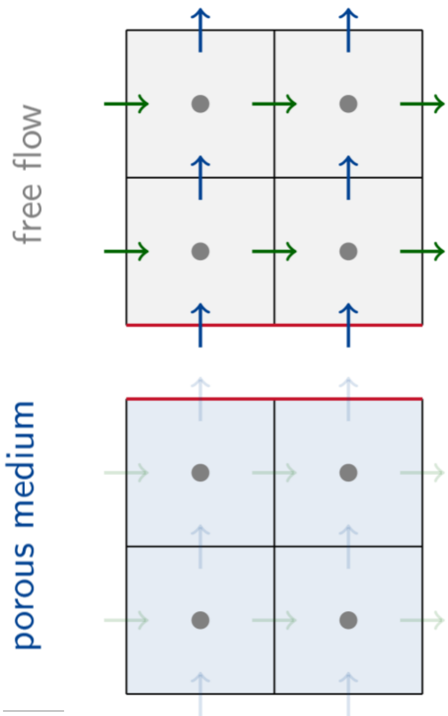
$$\left[\left(\varrho_\mathrm{g} h_\mathrm{g} \mathbf{v}_\mathrm{g} + h_\mathrm{g}^\mathrm{a} \mathbf{j}_{\mathrm{mass,eff}}^\mathrm{a} + h_\mathrm{g}^\mathrm{w} \mathbf{j}_{\mathrm{mass,eff}}^\mathrm{w} - (\lambda_\mathrm{g} + \lambda_\mathrm{t}) \nabla T\right) \cdot \mathbf{n}\right]^{\mathrm{ff,if}}$$
$$= -[(\varrho_\mathrm{g} h_\mathrm{g} \mathbf{v}_\mathrm{g} + \varrho_\mathrm{l} h_\mathrm{l} \mathbf{v}_\mathrm{l} - \lambda \nabla T) \cdot \mathbf{n}]^{\mathrm{pm,if}}$$

Implementation

Implementation

Spatial Discretization Scheme

staggered grid / CCFV

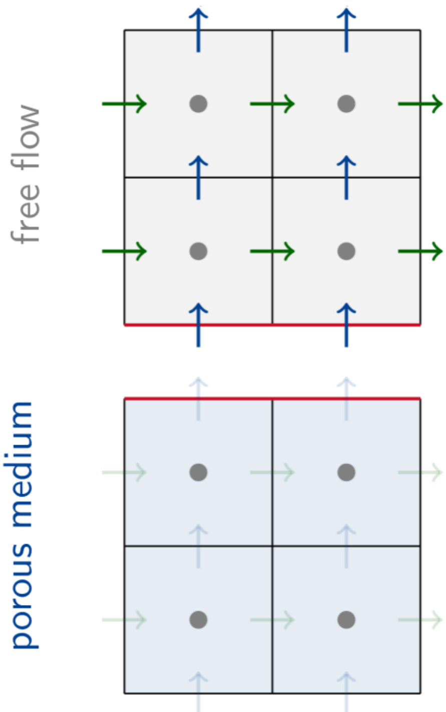


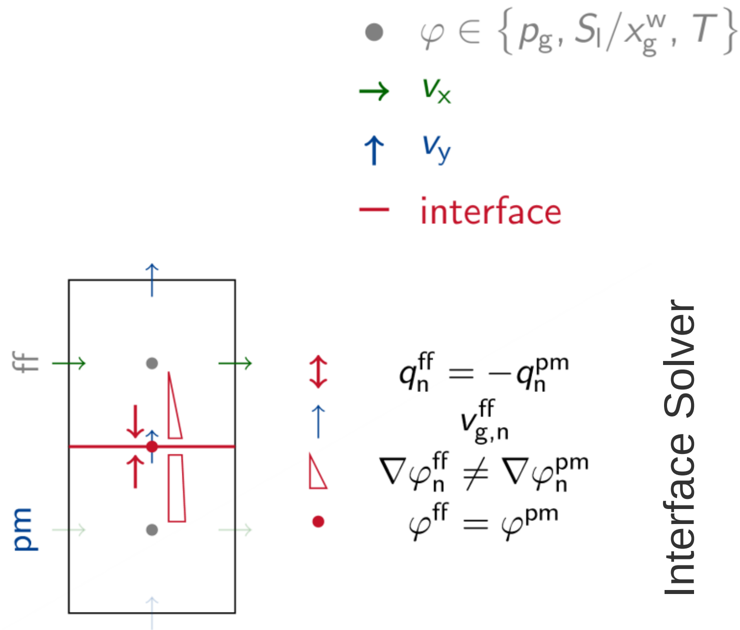
- ● $\varphi \in \{p_\mathrm{g}, S_\mathrm{l}/x_\mathrm{g}^\mathrm{w}, T\}$
- → v_x
- ↑ v_y
- — interface

Implementation

Coupling – Interface Solver

staggered grid / CCFV

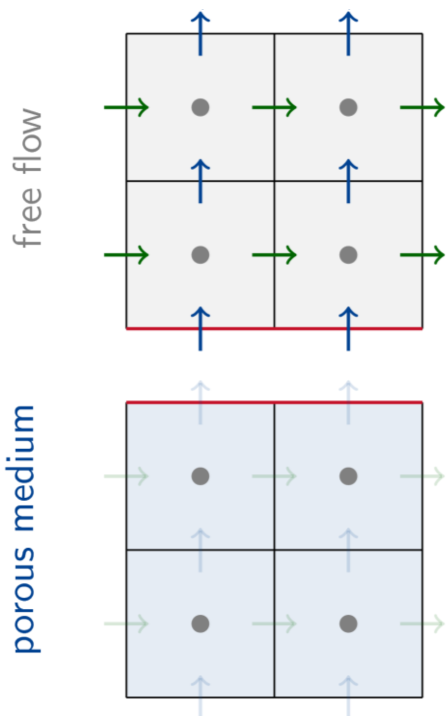


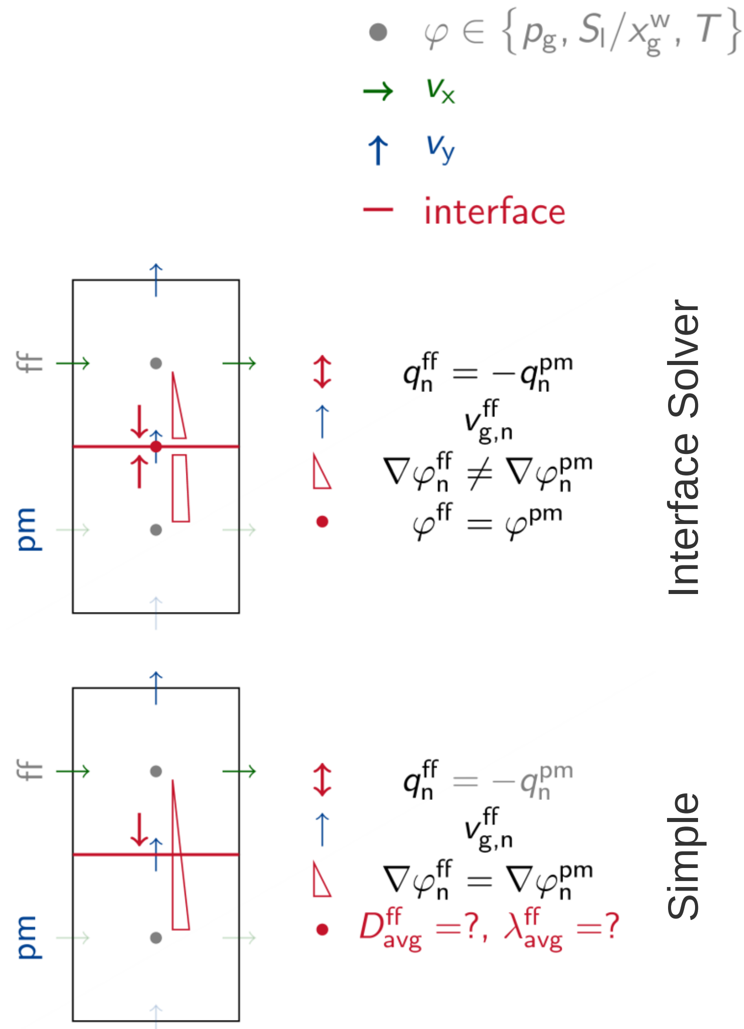


Implementation

Coupling – Simple

staggered grid / CCFV





Introduction
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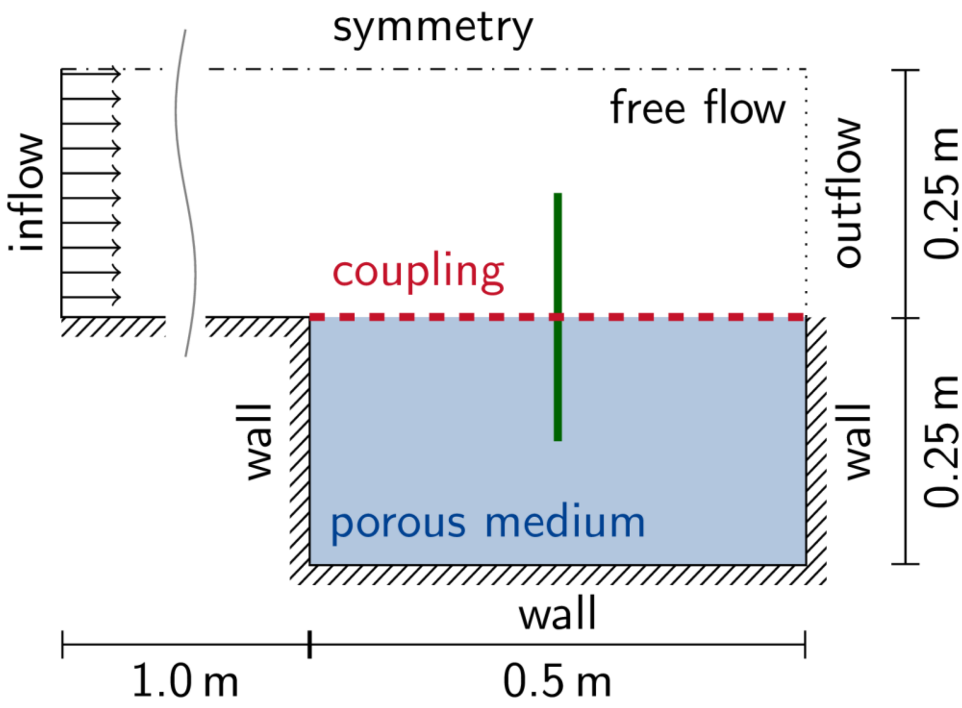
Implementation

Examples

Examples

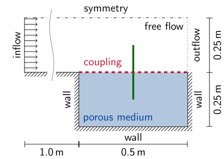
Basic Simulation Setup

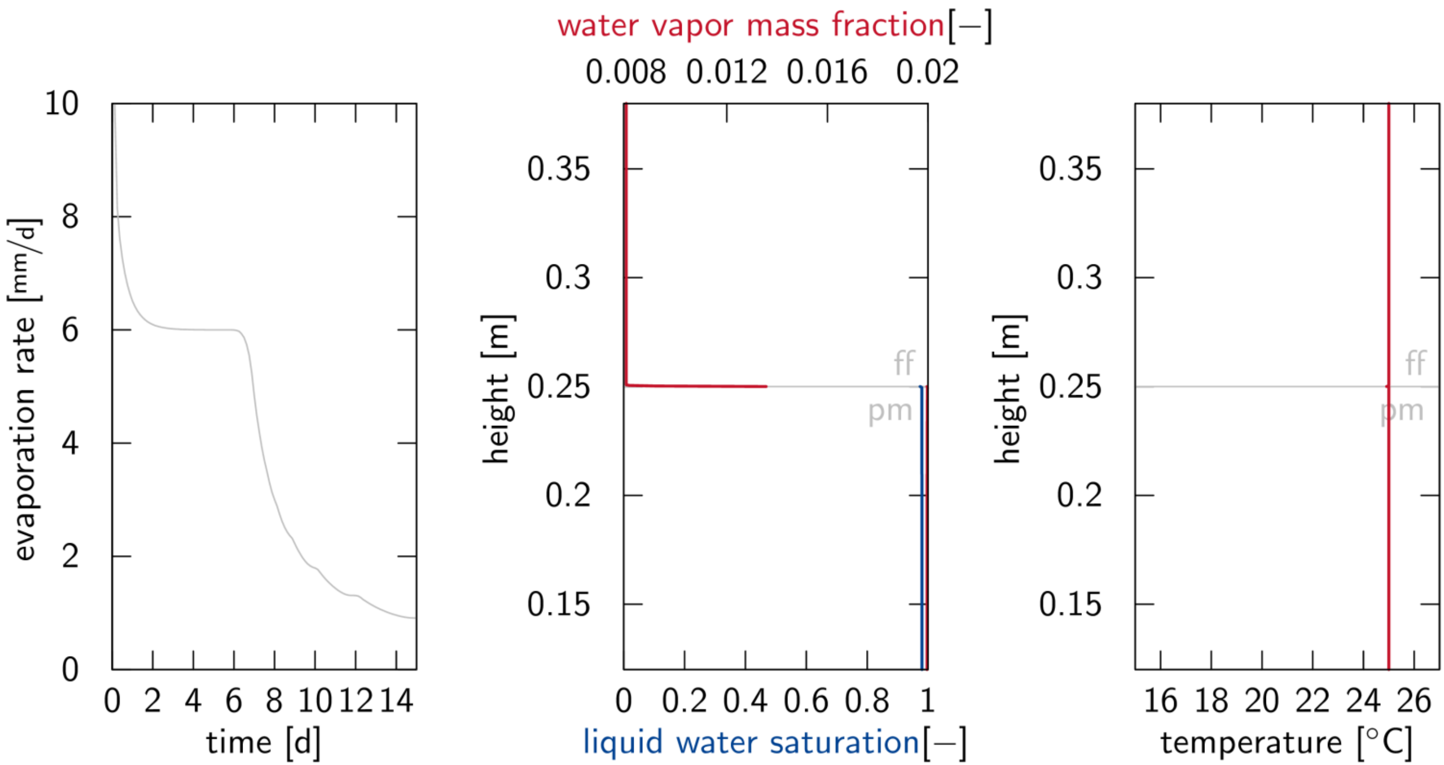
parameter	value
$\mathbf{v}_g^{ff}$ [m/s]	$(3.5, 0)^T$
p_g^{ff} [Pa]	1E5
$X_g^{w,ff}$ [–]	0.008
T^{ff} [K]	298.15
p_g^{pm} [Pa]	1E5
S_l^{pm} [–]	0.98
T^{pm} [K]	298.15



Examples

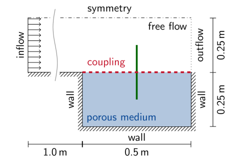
Typical Stages of Evaporation

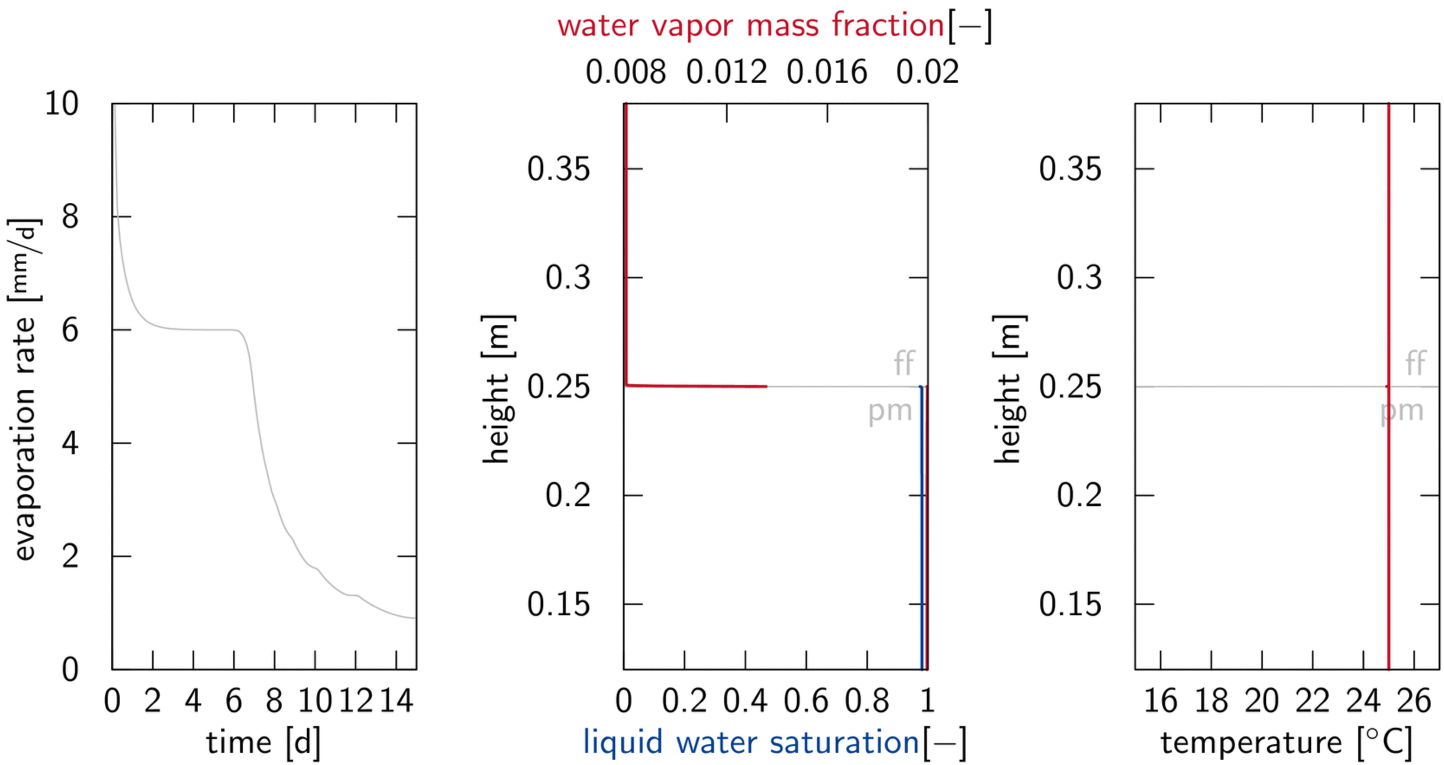




Examples

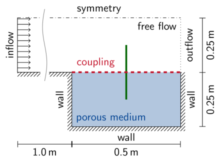
Typical Stages of Evaporation – Stage-I (Constant Rate)

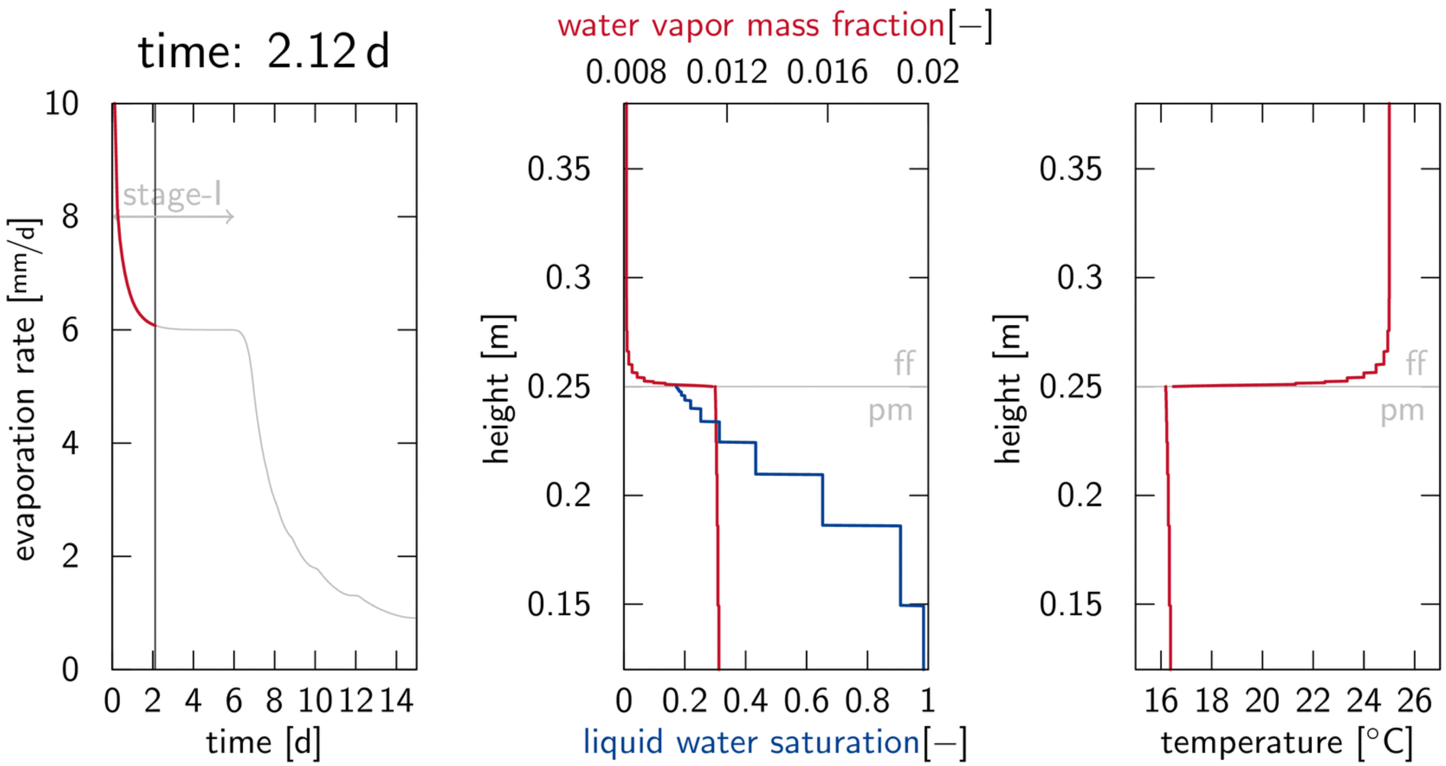




Examples

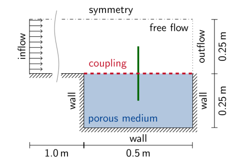
Typical Stages of Evaporation – Stage-I (Constant Rate)

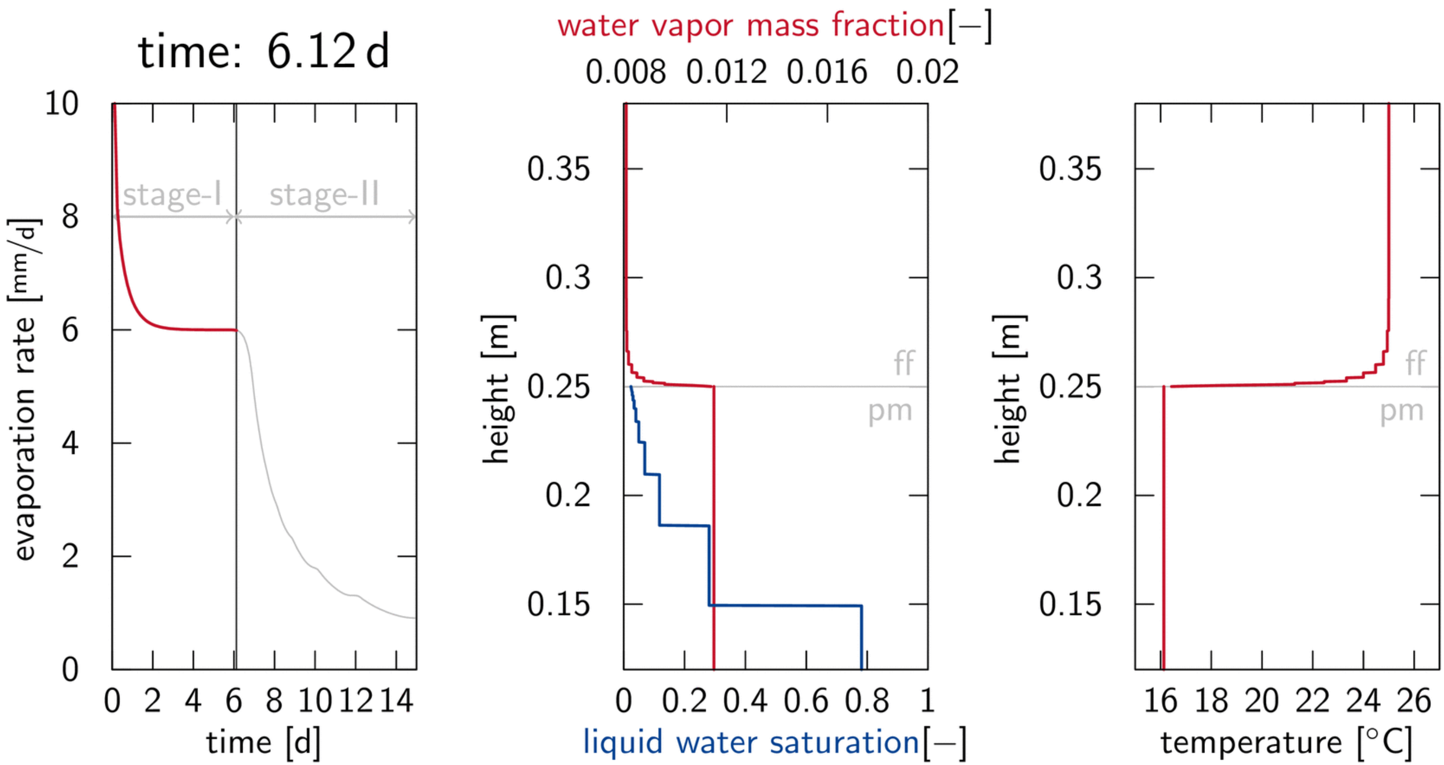




Examples

Typical Stages of Evaporation – Stage-II (Falling Rate)





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Vielen Dank!

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